Pergamon
www.elsevier.com/locate/jappmathmech
PII: S0021-8928(02)00126-0
J. Appl. Maths Mechs, Vol. 66, No. 6, pp. 859–865, 2002

0021-8928/02/$—see front matter

NIKOLAI GUR'YEVICH CHETAYEV (on the 100th anniversary of his birth)†

V. V. RUMYANTSEV

The 6 December 2002 marks the 100th anniversary of the birth of the great Russian scientist in the field of mechanics and teacher Nikolai Gur'yevich Chetayev, who made a huge contribution to the development of the theory of the stability of motion and analytical dynamics, was the founder and leader of the famous Kazan and then the world-renown Moscow School of General Mechanics, and was Editor-in-Chief of the journal *Prikladnaya Matematika i Mekhanika* from 1945 to 1959.

He was born in the village of Karaduli in the Laishevsk district of Kazan province into a sacristan's family, and received his secondary education at Kazan Third High School, renamed, after the revolution, the Second-Level School, which he left in 1919. He was an excellent pupil, and moved from class to class with commendable certificates of progress and good conduct; he was interested in natural sciences and the exact sciences. Having worked for a short time in a local government military registration and enlistment office, in 1920 he entered the Mathematics Section of the Department of Physics and Mathematics of Kazan University. He studied with inspiration, carrying along his peers, and he presided over the N. I. Lobachevskii study group. His industry and outstanding abilities drew him to the attention of the University professors. As a student, he entered into correspondence with Academician V. A. Steklov, on whose advice he began the in-depth study of the work of Lyapunov and Poincaré; he published his first scientific paper "Diffraction of light in opaque media".

†*Prikl. Mat. Mekh.* Vol. 66, No. 6, pp. 899–907, 2002.

Having graduated from University in 1924, he soon married Mariya Vasil'yevna Zhdanova, the sister of a school friend, and for about a year gave private lessons to support his family. At this time he solved the problem set by Steklov concerning the stellar dynamics of near-ellipsoidal figures of equilibrium of a rotating liquid. Lyapunov was known to have demonstrated the instability of pear-shaped figures of equilibrium, thereby refuting Darwin's cosmogonic hypothesis. In this context, the question again arose of the formation of twin stars. Chetayev proposed a model of a decaying star, introducing into consideration the forces of radiative contraction towards the centre of gravity at constant speed, and demonstrated the stability of figures of equilibrium as similar as desired to an ellipsoid of bifurcation [3, 4] (here and below, the numbers in square brackets correspond to the numbers from the list of publications by Chetayev given in the book: CHETAYEV, N. G., *Stability of Motion. Papers on Analytical Mechanics*. Izd. Akad. Nauk SSSR, Moscow, 1962).

In January 1926, Chetayev enrolled as a postgraduate student in the Department of Mechanics of Kazan State University (KSU) under Professor D. N. Zeiliger, a specialist in the field of kinematics and geometry, who laid down the principles of screw formalism. During his 3 years as a postgraduate student, Ghetayev carried out and published several important studies in which he reduced Poincaré's equations to canonical form and generalized them into systems with non-stationary constraints and dependent variables, to which he extended the classic results of Hamilton, Jacobi, and Poisson [5, 6], investigated stability in the Poisson sense [7, 8], and gave an analysis of equilibrium figures of a rotating liquid that were derivatives of ellipsoidal figures. Here he generalized two Poincaré theorems: the theorem of equilibrium curves passing through a bifurcation point, and the theorem of the law of change of stability [9]. These studies indicate that, from the very start of his scientific activity, Chetayev dared to tackle problems in the classics of science and obtained new, important results.

After his postgraduate studies, Chetayev, on the suggestion of D. N. Zeiliger, was sent, by Supreme Council of National Economy of the USSR for 1 year to work at the Hydroaerodynamic Institute of Göttingen University. There he became acquainted with the leading aerodynamic school of Prandtl, took part in the work of Prandtl's seminar, and continued his scientific research. In particular, using Lyapunov's method he obtained the conditions of stability and instability of normal aircraft flight [10].

Having returned from Göttingen in March 1930 Chetayev was appointed Assistant Professor at Kazan State University, and in September of the same year he was appointed Professor of Mechanics, a post that had become vacant after Professor D. N. Zeiliger left Kazan. It should be pointed out that, in 1927, Zeiliger had organized a group of students specializing in the field of aerodynamics. Chetayev continued to organize aviational education in Kazan, and, on his initiative, in 1931 the Aerodynamics Department was created, which at the start of 1932 became the independent Kazan Aviation Institute (KAI). Chetayev, appointed Deputy Director of the Institute, supervised its organizational, educational, and scientific work. He defined the profile of the Institute and planned the first years of the education and provision of students with practical experience at aircraft factories and in scientific research institutes. Special attention to, and special demands on, the quality of the general fundamental training of students became traditional at the Institute. In 1934, preparations were made to construct a wind tunnel, which was designed under the supervision of Chetayev and G. V. Kamenkov. The use of this wind tunnel for aerodynamic research played an important role during World War II.

In the 1930s, the well-known Vermutungsseminar, organized and run by Chetayev, became a centre bringing together almost all the creative pure and applied mathematicians in Kazan. Its meetings made an intensive study of the famous "General problem of the stability of motion" of Lyapunov and presented the results of research by participants at the seminar. The themes of the seminar were dictated by problems of the stability of motion, analytical dynamics, and qualitative methods of differential equations. Besides Chetayev's postgraduate students, scientists (then assistant professors) I. G. Malkin and K. P. Persidskii, and also corresponding member of the USSR Academy of Sciences N. G. Chebotarev, took an active part in the work of the seminar. Reports of work in progress by authors, with acknowledgement of the difficulties arising, gave the meetings a special air of controversy, mutual assistance, and, in addition, scientific rivalry, which Chetayev was able tactically, as arbitrator, to steer in a healthy direction.

Thus, an area of research was created in mechanics that subsequently came to be called the Kazan, and later, the Moscow Chetayev School of stability theory and analytical mechanics.

In the work of participants at the seminar, published in *Proceedings of the KSU*, *News of the Kazan Physics and Mathematics Society*, and *Proceedings of the KAI*, the mathematically rigorous Lyapunov theory was further developed, and its great practical value was revealed. The development of science and engineering showed how important it was to foresee the need for research in the new field of mechanics, the value of which remained unclear, and to organize the start of this research and to attract young mathematicians to it.

stable

The research by Lyapunov on the theory of the stability of motion got ahead of its time. In his lifetime there were no apprentices or successors in this field of science, and Lyapunov's stability theory for a long time not only underwent no development but also was not seriously applied to any extent. Chetayev's great contribution was to reveal the immense importance of Lyapunov's work by subsequent creative development of its profound ideas, the philosophical substantiation of the theory, and the demonstration of its importance for applications in physics and engineering. Chetayev, it seems, was one of the first to understand the physical basis of Lyapunov's theory and to appreciate its fundamental importance and its technical possibilities. At the start of the 1930s, he established his general theorem of the instability of motion and obtained the first post-Lyapunov results on inversion of the famous Lagrange theorem on the stability of equilibrium, and he organized wide-ranging research on the development of the theory and its use to solve important technical problems. He thus became essentially the scientific successor to Lyapunov, continuing his work.

Chetayev firmly believed that problems of stability were of fundamental and practical importance, and that the laws of nature were closely related to them. He asked: how are the laws of nature established? Any time we come to study a specific mechanical effect, we must remember the existence of unaccounted for small forces and inaccuracy in the determination of initial data. We take into account only basic motive forces, i.e. fundamentally it is a different problem that is being solved, and the question is, when do our theoretical motions reflect reality [19]?

Small disturbing forces and small deviations in the initial data have an effect on the motions of mechanical systems, in some cases a small effect and in others a considerable effect. And it is clear that only the first (stable) motions are important for the creation of the real picture of natural phenomena.

The system of our physical knowledge has been created through the ages in a quite definite manner. To explain a particular effect, hypotheses are put forward concerning the forces acting in this phenomenon, which leads to certain differential equations describing the phenomenon. If, according to a particular hypothesis, the values of certain functions Φ_m lie close to the observed values of these functions Φ_n, and if the deviations $\Delta_{mn} = |\Phi_m - \Phi_n|$ are small and remain within the limits of experimental error, then this hypothesis is adopted as a law of nature. However, if at least one of these differences Δ_{mn} is, in some case, always greater than the permissible experimental error, the hypothesis is rejected and an attempt is made to create a new hypothesis where these differences will always be smaller than experimental errors. And if all our accurate natural science is built upon laws giving small values of the differences Δ_{mn}, then, according to the definition of Lyapunov stability, this may be when theoretical motion is stable in relation to the functions Φ_m.

This does not mean that unstable motions cannot occur in nature. It means only that our ideas concerning nature rest largely on motions that are stable to a particular degree in relation to the functions observed.

This assertion, which is a simple consequence of determining stable undisturbed motions and the requirement of small deviations of the results of theory from experimental data, and which relates more to the structure of our scientific knowledge, Chetayev termed the postulate of stability. He thereby established a new scientific principle consisting of the acceptance, outside the experimental laws of nature, of the theoretical properties of stability as a necessary requirement of small deviations of theory from experiment in the functions observed.

For specific laws of physics, Chetayev [20] demonstrated the correctness of the postulate of stability, namely: (1) Hooke's law in the statics of an elastic medium; (2) the behaviour of entropy S: $dV|dt \geqslant 0$, $V = S - S_0$, where $S_0 = \max S$; (3) Kepler's laws of motion of celestial bodies; (4) Torricelli's law of the lowest position of the centre of gravity of a system of heavy bodies in equilibrium; (5) Newton's law of universal gravity and Coulomb's law in electricity.

Examining the motion of a conservative system in a field of prescribed forces with force function $U_0(q_1, \ldots, q_n)$ and perturbing forces with an unknown force function $W(q_1, \ldots, q_n)$, the influence of which on the motion at an arbitrary point q_s is proportional to the density of trajectories at this point $A^2 = \psi\psi^*$, Chetayev showed [12] that perturbing forces to a relatively smaller degree disturb motions for which the integral $\int W\psi\psi^* d\tau$ takes the maximum value. Hence, he obtained the basic equation of stable orbits, which, for a free point mass, has the form of Schrödinger's differential equation upon which his quantum mechanics is based.

Chetayev's general theorem of the instability of motion plays an important role in the development of the method of Lyapunov functions. Having begun work on the inversion of Lagrange's theorem of the stability of equilibrium, he discovered that both Lyapunov theorems of the instability of motion are relatively incomplete in nature, and he proposed a new theorem of instability [11] in which, along with the Lyapunov function V and its first derivative with respect to time $\dot{V}$, the second derivative $\ddot{V}$ occurs. Then he proved a theorem [16] in which use is made of two functions V and W and their first derivatives with respect to time, and, finally, he gave a general theorem [23] on the instability of motion with one function V and its derivative $\dot{V}$, from which both Lyapunov theorems of the instability of motion follow as special cases .

Subsequently, N. N. Krasovskii and I. Vrkoč proved the inversion of this Chetayev theorem, thereby substantiating its universality. It played an important role both in the work of Chetayev himself and in the work of his pupils and successors, and now, along with Lyapunov's theorem on stability and asymptotic stability, it forms the basis of the methods of Lyapunov functions.

In a series of studies Chetayev investigated the problem of the inversion of Lagrange's theorem. Initially he proved [11] the instability of an isolated equilibrium position using Kronecker characteristics. The complexity of this solution prompted him to search for more elementary solutions, which were found and published in [16, 23, 48]. In the first of these [16] he proved instability if U is a homogeneous force function U_m of order m and can take positive values. In the second of these papers [23] he gave a new proof of the inversion of Lagrange's theorem for the general case of an analytical force function without a maximum in an isolated equilibrium position. Here, Chetayev abandoned a direct search for the function V and used the complete integral of the Hamilton–Jacobi equation $H(q, h, \alpha)$, showing that it satisfies the conditions of his theorem of instability. An elementary proof was given for two cases, (1) when U is a homogeneous function of order m and (2) when $U = U_m + U_{m+1} + \dots$, and a positive sign of the functions U and $mU_m + (m + 1)U_{m+1} + \dots$ is determined from lower-order terms U_m without any need to consider higher-order terms. The third of these papers [48] contains an elementary proof for a number of other, more complex and finer cases of inversion of Lagrange's theorem.

A large number of studies by Chetayev were devoted to Gauss' principle and its variations. Here, above all it must be remembered that, when examining non-linear non-holonomic constraints, Appell and Delassus came to the conclusion that the d'Alembert–Lagrange principle is incompatible with Gauss' principle of least constraint. This fact prompted Chetayev to provide a generalization of the principal concept of analytical mechanics – the concept of virtual displacements, with which these two principles proved to be compatible.

He also put forward the new idea of the freeing of material systems from constraints. Before this, repeated attempts had been made to generalize the Gaussian concept of freeing and at the same time Gauss' principle. Chetayev proposed that the term "freeing of the system" be applied to any of its transformations obeying a certain mathematical algorithm (parametric freeing). With the definition given to them, from the d'Alembert–Lagrange principle, if it is introduced as a consequence of the axiom of definition of smooth constraints, Gauss' principle and its generalization follow.

In 1936, Chetayev's wife died of tuberculosis. Her death was a heavy blow to him and his son. Four years later he married Vera Aleksandrovna Samoilova.

In 1940, Chetayev was invited, at the instigation of Chaplygin, to work in the USSR Academy of Sciences. In October 1940 he moved to Moscow, set up the Department of General Mechanics of the Institute of Mechanics of the USSR Academy of Sciences, founded in 1939, and, up to the end of his life, ran this department. At the same time he was a Professor at Moscow State University (MSU). In August 1944 he became Deputy Director and from 1945 to 1953 he was Director of the Institute of Mechanics of the USSR Academy of Sciences. In the final years of his life, he ran the MSU Department of Theoretical Mechanics In September 1943 he was elected a Corresponding Member of the USSR Academy of Sciences, specializing in mechanics, and in 1947 he was elected a Full Member of the Academy of Artillery Sciences.

His scientific and public activity in the Moscow period of his life was very diverse and extremely fruitful. In the years when he was its Director, the Institute of Mechanics flourished, gained considerable authority, and became a leader in its field. Chetayev possessed a great talent for organization, led with great wisdom and tact, managed to recruit prominent scientists and young talent, thereby creating conditions for the development of all leading departments of the Institute, and supported all that was at the leading edge and progressive. He tried to justify the need to concentrate efforts on developing promising theoretical research and fought against any narrow thinking in planning. The staff of the Institute had great respect for him.

Chetayev was exceptionally creative in this period of his life. In 1940–1959 he wrote over 40 papers and books, including the monograph *The Stability of Motion*, which has now been published in five editions (1946, 1955, 1962, 1965, and 1990).

He obtained fundamental results in the area of analytical dynamics. He proposed an original version of Gauss' principle [25]. Estimating, for a mechanical system with linear constraints, the work T_μ in an elementary cycle consisting of forward Gaussian motion in a field of active forces and reverse motion in a force field that would be sufficient to produce real motion if the system were completely free, Chetayev, by using Gauss' principle, demonstrated that the work in a similar cycle for real motion is the maximum work T_μ. This theorem is equivalent to Gauss' principle. It makes it possible to broaden the nature of the mechanical systems normally examined by enlisting Carnot's principle from thermodynamics.

Chetayev's work *On Certain Constraints with Friction* [67] is of considerable interest. By the addition of friction forces to prescribed forces, systems with friction normally lead to systems with smooth constraints. Chetayev showed that, with fairly broad assumptions concerning friction forces, the application of Lagrange's method makes it possible, for mechanical systems under constraints with friction, to establish a general principle without reactions of constraints occurring in it explicitly.

Another group of studies was devoted to the theory of Poincaré equations, which were mentioned above. He developed this theory further [26]. A method was proposed for constructing groups of virtual and real displacements where the constraints are prescribed in differential form. An important concept was introduced concerning cyclic displacements of the system, and its use for reducing the order of equations and their integration was demonstrated. The possibility of solving the Hamilton–Jacobi equation in more general functions than the action function was also pointed out. With this work, Chetayev essentially created a new division of analytical mechanics – the dynamics of systems in group variables, and in many ways he determined the direction of future research in mechanics.

Chetayev proved the fundamental theorem that, if unperturbed motion is stable, then all characteristic exponents in equations in Poincaré variations are equal to zero, are correct and reducible, and have a sign-definite quadratic integral. The value of this theorem lies, in particular, in the fact that it serves as the basis of an effective method he proposed for constructing Lyapunov functions in the form of a linear bundle of first integrals of equations of perturbed motion.

These results enabled him to give an extension of the optical-mechanical analogy, in which he always showed great interest, stressing that "the roots of the excellent results found in analytical dynamics after Lagrange lay in the analogy between mechanics and optics. For contemporary problems, this analogy, in my view, plays no less a role". In his paper "Klein's problem. On the optical-mechanical analogy" [65], published posthumously, Chetayev showed that the group of unimodal linear transformations corresponding to stable unperturbed motion is represented in the complete group of Lorentzian transformations, which is the basis for the Cauchy and Maxwell theories of light that appeared after Huygens' theory. Thereby, a solution was given of Klein's problem concerning the further development of the optical-mechanical analogy within the framework of analytical mechanics. The excellent completion of this series of studies is Chetayev's paper "The extension of the optical-mechanical analogy" [61] – the last paper published in his lifetime.

A large number of his papers were devoted to investigating the stability of unsteady motions in a first approximation, and also to obtaining estimates of the solutions of a system of differential equations. Using the method of Lyapunov functions he demonstrated theorems on stability and instability to a first approximation both for regular and irregular systems. He established an important theorem concerning the least characteristic exponents of an unsteady system, the first-approximation coefficients of which tend to be constant as time increases without limit.

In a number of papers on the application of the method of Lyapunov functions to the problem of stability, Chetayev demonstrated the effectiveness of this method and also confirmed the possibility of calculating estimates of the quality of the transients in the system. This method of estimates using quadratic Lyapunov functions has been widely used in work by many researchers.

In the development and application of Lyapunov's theory of stability and, in particular, the method of Lyapunov functions, Chetayev was undoubtedly foremost among scientists of the twentieth century. From the 1930s onwards, he frequently stressed the possibility and effectiveness of using Lyapunov's method to solve important technical problems of stability. In his scientific creativity, applied problems of the theory of stability, such as the problem of the stability of rotational motions of a missile, the stability of a rigid body with a fluid-filled cavity, a gyroscope in a gimbal suspension, the motion of a vibratory mill, and many others, occupy an important place. In particular, he was the first to publish a rigorous solution in a non-linear formulation of the problem of the stability of a missile with a cavity completely filled with an ideal fluid performing irrotational motion. He attacked great importance to the solution of this problem, since in a number of cases, on the basis of the given solution, it is possible to indicate an adequate stability margin against unaccounted for negative influences of viscosity. He began a series of investigations on the stability of the motions of a rigid body with cavities partially or completely filled with an ideal or viscous fluid.

Ending this brief review of Chetayev's investigations, it is necessary to dwell on his monograph *The Stability of Motion* (1946) and the posthumously published textbook *Theoretical Mechanics* (1987).

The first of these – a small book – contains investigations of the stability of the motion of mechanical systems with a finite number of degrees of freedom, which were begun by the classical work of Lyapunov and continued by Russian scientists, and consist of the systematic application of Lyapunov's second method. Fundamental results in this area were obtained. Here, he makes no attempt at a complete exposition of all his results but confines himself to those investigations that are of greatest value for applications. In the second edition (1955), he managed, while retaining the size of the book, to include new theoretical results and a number of new problems to illustrate the theory.

In the book, the condition of asymptotic stability is proposed, which is slightly more general than the condition corresponding to Lyapunov's theory. An examination is made of the influence of perturbing forces on equilibrium, and Kelvin's theorems concerning the influence on stability of dissipative and gyroscopic forces are rigorously proved. The established possibility of estimates of characteristic exponents by averaging the coefficients is of considerable importance for practical calculations.

Chetayev's monograph became a reference book for scientists and engineers working in the field of stability and its applications, and also became a textbook for students. It has been translated into many foreign languages, in China, the United States and other countries. This book, like his entire scientific activity, helped bring stability problems to the attention of a wide circle of researchers. It was after the publication of the first edition of the book (1946) that a large number of studies of stability, increasing year on year, began to appear in the literature. It is generally recognized that, in the area of the theory of the stability of motion, Russian scientists occupy a leading place in world science. Huge credit in this regard should go to Chetayev.

In the last years of his life, Chetayev worked on writing the textbook *Theoretical Mechanics* for university students. Lectures on theoretical mechanics that he gave over many years for students at the KSU, the KAI, and the MSU had a great influence on the teaching of theoretical mechanics, including many courses published subsequently. He based his course on classical traditions dating back to Lagrange, and creatively development these traditions. The high mathematical level of exposition in his course was always combined with a large number of stylishly selected problems generally having practical application. As a great scientist and excellent teacher, he frequently included in his general course a discussion of problems on which he happened to be working at the time.

From 1945 until the end of his life, Chetayev was the Editor-in-Chief of the journal *Prikladnaya Matematika i Mekhanika*. Here, he again gained universal respect for his adherence to principles in judgements and for his objectivity in evaluations. That *Prikladnaya Matematika i Mekhanika* became such an authoritative journal on mechanics both in Russia and abroad is largely due to Chetayev.

His unexpected death from heart disease occurred on 17 October 1959 in Moscow.

His scientific and public activity was remarkable not only for his personal scientific creativity and fruitful ideas, which were abundant in his work, but also for the education and training of a large number of highly skilled engineers, teachers, and scientific workers.

In the Moscow period of his life, Chetayev lectured at Moscow University. Here he gave a series of general and special courses on analytical mechanics, the theory of the stability of motion, and the theory of relativity. By the way, Chetayev showed a constant and keen interest in theoretical physics and quantum mechanics, and many notes in his notebook indicate how finely he understood contemporary problems of science. He enthusiastically led many student and postgraduate seminars at which he himself often presented papers, very generously shared his ideas and knowledge, was able to foster the interest and taste of young people in science, and carefully encouraged independent scientific research. And young people responded to this with enthusiasm and deep respect.

Chetayev paid a great deal of attention to the training of young scientists on postgraduate and doctoral levels. Under his leadership, a large number of people were trained and defended Master's and Doctoral dissertations: G. V. Kamenkov, S. G. Nuzhin, G. N. Fedorov, P. A. Kuz'min, M. Sh. Aminov, V. V. Rumyantsev, V. M. Starzhinskii, N. N. Krasovskii, B. S. Razumikhin, S. V. Kalinin, A. A. Bogoyavlenskii, G. K. Pozharitskii, Sh. S. Numganova, K. Ye. Shurova, S. F. Saikin, V. I. Kirgetov, and many others. Chetayev required his students to show persistence in mastering science and recommended that they work hard and study the classics of science. Generally, he himself did not specify the themes of the dissertations of his pupils but directed their attention to the importance of particular weighty problems in mechanics and advised them to try their powers in solving them. In the course of these attempts, specific themes for dissertations normally arose. A great scientist, he possessed a happy gift of spotting the capabilities of people, developing these capabilities, giving people confidence in their powers, and fostering an independence of scientific thinking. He astounded, but was never overbearing, with his enormous erudition (and he knew and understood a remarkable amount), did not prompt finished formulations of the solution of a particular problem, but gradually and imperceptibly, by long and patient discussions, indicated the correct, most promising path for solving a problem so that, as a result, pupils gained confidence in their capabilities and powers and the satisfaction that they themselves had solved the problem.

Chetayev always emphasized the rigorous formulation of new problems in mechanics and the creation of rigorous methods for solving them. At the same time, he fought against excessive generalization of mechanics problems, which sometimes deprived them of any interest, and was against unjustified hypotheses, often introduced during the course of the solution of a problem and motivated only by the blind desire for its "solution". This purity came from the legacy of Lyapunov, which Chetayev himself always followed and which he tried to pass on to his pupils, but not only to his pupils. Everyone knows the inestimable importance to science of Chetayev's presentations at congresses and conferences on mechanics and mathematics, and at seminars, and his observations on particular papers and presentations. Here he always spoke directly, often abruptly, without respect of persons, being guided only by the interests of science. The benefit from his observations were enormous, and correct formulations of problems emerged and ways of solving them became clear. The formulation of the problem of stability for systems with an after effect can be taken as an example. His personal scientific creativity and his outstanding qualities as a scientist, organizer, and person gave him indisputable authority within science.

A very great scientist and classical scholar of the twentieth century, Chetayev was a fine man with a sensitive, pure spirit, was responsive and benevolent towards people and very accessible, and was scrupulously honest and principled. He had a highly developed sense of honour and inner dignity and exceptional modesty. He was a great patriot, with a passionate love of his country, and was a shining example of a remarkable man and great scientist who brought glory to Russian science.

His classical works have entered for ever the treasure-house of science and will be rich source of new fruitful ideas for subsequent generations of scientists.

Translated by P.S.C.